Evolutionary Mechanisms: Problems Set 2 Answer Key

1. Suppose you have a population of 3 individuals that is variable at a locus with two alleles. Suppose additionally that there are 2 AA individuals and 1 aa individual. For each of the next 5 generations, calculate the probability that the allele frequency of the A allele will be 0, 0.167, 0.333, 0.5, 0.667, 0.833, or 1.0 if there is no selection acting.

Let

$T_{j,i}$ = Pr(going from i to j copies in one time step)

θ_i= Pr(Population has i copies at time t)

The Wright-Fisher model states that

$$T_{j,i} = \binom{N}{j}(p)^j(1-p)^{N-j} \text{ where } p = \frac{i}{N}$$

Thus the population transition matrix is

$$\mathbf{T} = \begin{pmatrix} T_{0,0} & T_{0,1} & \cdots & T_{0,6} \\ T_{1,0} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ T_{6,0} & \cdots & \cdots & T_{6,6} \end{pmatrix} = \begin{pmatrix} 1 & 0.3349 & 0.8779 & 0.0156 & 0.0014 & 0.00002 & 0 \\ 0 & 0.4019 & 0.2634 & 0.0937 & 0.0165 & 0.0006 & 0 \\ 0 & 0.2009 & 0.3292 & 0.2344 & 0.0823 & 0.0080 & 0 \\ 0 & 0.0536 & 0.2195 & 0.3125 & 0.2195 & 0.0536 & 0 \\ 0 & 0.0080 & 0.0823 & 0.2344 & 0.3792 & 0.2009 & 0 \\ 0 & 0.0006 & 0.0165 & 0.0937 & 0.2634 & 0.4019 & 0 \\ 0 & 0.00002 & 0.0014 & 0.0156 & 0.8779 & 0.3349 & 1 \end{pmatrix}$$

and the population vector at time t is

$$\vec{\boldsymbol{\theta}}_t = \begin{pmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_6 \end{pmatrix} \text{ where } \vec{\boldsymbol{\theta}}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Since, $\vec{\boldsymbol{\theta}}_n = \mathbf{T}^n \ \vec{\boldsymbol{\theta}}_0$,

$$\vec{\boldsymbol{\theta}}_1 = \begin{pmatrix} 0.0014 \\ 0.0165 \\ 0.0823 \\ 0.2195 \\ 0.3292 \\ 0.2634 \\ 0.8779 \end{pmatrix}, \quad \vec{\boldsymbol{\theta}}_2 = \begin{pmatrix} 0.0180 \\ 0.0545 \\ 0.1111 \\ 0.1739 \\ 0.2197 \\ 0.2145 \\ 0.2084 \end{pmatrix}, \quad \vec{\boldsymbol{\theta}}_3 = \begin{pmatrix} 0.0490 \\ 0.7119 \\ 0.1081 \\ 0.1413 \\ 0.1658 \\ 0.1622 \\ 0.3024 \end{pmatrix}, \quad \vec{\boldsymbol{\theta}}_4 = \begin{pmatrix} 0.0848 \\ 0.7312 \\ 0.0980 \\ 0.1168 \\ 0.1298 \\ 0.1239 \\ 0.3737 \end{pmatrix}, \quad \vec{\boldsymbol{\theta}}_5 = \begin{pmatrix} 0.1199 \\ 0.0684 \\ 0.0860 \\ 0.0970 \\ 0.1036 \\ 0.0966 \\ 0.4285 \end{pmatrix}$$

2. In the annual morning glory, *Ipomoea purpurea*, the average selfing rate, as measured by several experiments is about 0.7. In one local population, the frequencies of two alleles at the *Esterase-1* locus are 0.65 and 0.35. Calculate the expected genotype frequencies in this population.

Assuming that the population is at its equilibrium,

$$\hat{p}_{11} = p_1^2 + \left(\frac{s}{2-s}\right)p_1 p_2 = (0.65)^2 + \left(\frac{0.7}{2-0.7}\right)\times 0.65\times 0.35 = 0.545$$

$$\hat{p}_{12} = 2(1-s)\left(\frac{2}{2-s}\right)p_1 p_2 = 2(1-0.7)\left(\frac{2}{2-0.7}\right)\times 0.65\times 0.35 = 0.21$$

$$\hat{p}_{22} = 1 - \hat{p}_{11} - \hat{p}2 = 1 - 0.755 = 0.245$$

3. Suppose there are two populations, one with 1000 individuals and one with 2000 individuals. The frequency of allele A in the first population is 0.5 and in the second population is 0.8. What will happen to allele frequencies in each population if the populations exchange migrants? Why?

The gene frequencies between the two populations will approach one another . Because the two populations are exchanging migrants the gene frequency in each of the mating pools will be a weighted mean of the two populations.

The rate at which they approach one another will be determined by the number of migrants exchanged by the two populations. Regardless of the migration rate at equilibrium $p_I = p_{II}$, and ignoring drift, we would expect the gene frequency to be

$$\frac{1000\times 0.5 + 2000\times 0.8}{3000} = \frac{2100}{3000} = 0.7$$ for both populations.

4. Show that the following are equilibrium genotype frequencies for the mixed-mating model with one locus with two alleles:

$$\hat{p}_{11} = \frac{s}{(2-s)}\hat{p}_1\hat{p}_2 + \hat{p}_1^2$$

and

$$\hat{p}_{12} = \frac{4(1-s)}{(2-s)}\hat{p}_1\hat{p}_2$$

Given the recursion equations,

$$p_{11}' = s \times p_{11} + \frac{s}{4} p_{12} + (1-s) p_1^2$$

$$p_{12}' = \frac{s}{2} p_{12} + 2(1-s) p_1 p_2$$

at equilibrium,
$p_{11}' = p_{11} = \hat{p}_{11}$ and $p_{12}' = p_{12} = \hat{p}_{12}$

Thus it follows that,

$$\hat{p}_{12} = \frac{s}{2} \hat{p}_{12} + 2(1-s) \hat{p}_1 \hat{p}_2$$

$$\Rightarrow \hat{p}_{12} - \frac{s}{2} \hat{p}_{12} = 2(1-s) \hat{p}_1 \hat{p}_2$$

$$\Rightarrow \hat{p}_{12}\left(1 - \frac{s}{2}\right) = 2(1-s) \hat{p}_1 \hat{p}_2$$

$$\Rightarrow \hat{p}_{12} = \frac{2(1-s)}{\left(1 - \frac{s}{2}\right)} \hat{p}_1 \hat{p}_2$$

$$= \frac{4(1-s)}{(2-s)} \hat{p}_1 \hat{p}_2$$

and

$$\hat{p}_{11} = s \times \hat{p}_{11} + \frac{s}{4} \hat{p}_{12} + (1-s) \hat{p}_1^2$$

$$\Rightarrow \hat{p}_{11} - s \times \hat{p}_{11} = + \frac{s}{4} \hat{p}_{12} + (1-s) \hat{p}_1^2$$

$$\Rightarrow \hat{p}_{11}(1-s) = \frac{s}{4} \hat{p}_{12} + (1-s) \hat{p}_1^2$$

$$\Rightarrow \hat{p}_{11} = \frac{s}{4(1-s)} \hat{p}_{12} + \hat{p}_1^2$$

$$= \frac{s}{4(1-s)} \frac{4(1-s)}{(2-s)} \hat{p}_1 \hat{p}_2 + \hat{p}_1^2$$

$$= \frac{s}{(2-s)} \hat{p}_1 \hat{p}_2 + \hat{p}_1^2$$